1 DSE 2021 Paper 1 Solutions

1.

$$\begin{aligned}(\alpha\beta^3)(\alpha^{-2}\beta^4)^5 &= (\alpha\beta^3)(\alpha^{-10}\beta^{20}) \\ &= \alpha^{-9}\beta^{23} \\ &= \frac{\beta^{23}}{\alpha^9}\end{aligned}$$

2. $\dfrac{4-3a}{b} = 5$
$4-3a = 5b$
$3a = 4-5b$
$a = \dfrac{4-5b}{3}$

3. (a) $(2x-y)(3x+2y)$.
(b)

$$\begin{aligned}8x-4y-6x^2-xy+2y^2 &= 4(2x-y)-(2x-y)(3x+2y) \\ &= (2x-y)(4-3x-2y)\end{aligned}$$

4. (a) $\dfrac{7(x-2)}{5}+11 > 3(x-1)$ and $x+4 \geq 0$

$8x < 56$ and $x \geq -4$

$-4 \leq x < 7$

(b) 6.

5. Let x and y be the number of stickers owned by the boy and the girl.

$$\begin{cases} x = 3y \\ 2(x-20) = y+20 \end{cases}$$

Substituting the 1st equation into the second gives: $2(3y-20) = y+20$.
Solving gives $x = 36$ and $y = 12$.
Therefore, the total number of stickers owned by the boy and the girl is $36+12$, which is 48.

6. Let $\$x$ be the marked price, then the cost $= \$(x-80)$
$x(1-10\%) = (x-80)(1+30\%)$

Solving gives $x = 260$
Therfore, the marked price of the shirt is \$260.

7. (a) $\angle POQ = 140° - 80° = 60°$

(b) Note that POQ is an equilateral triangle.
Thus, $r = 21$.

(c) Perimeter $= 3(21) = 63$

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8. (a)

$$\angle AEC = \angle DEB \quad \text{(common angle)}$$
$$\angle CAE = \angle BDE \quad \text{(given)}$$

$$\begin{aligned}\angle ACE &= 180^\circ - \angle CAE - \angle AEC \quad (\angle \text{ sum of } \triangle)\\ &= 180^\circ - \angle BDE - \angle DEB \quad \text{(proved)}\\ &= \angle DBE \quad (\angle \text{ sum of } \triangle).\end{aligned}$$

Hence $\triangle ACE \sim \triangle DBE \quad (A.A.A.)$

(b)(i) Since $AC^2 + AE^2 = 25^2 + 60^2 = 65^2 = CE^2$. $\triangle ACE$ is a right-angle triangle by the converse of Pythagorean's theorem.

(b)(ii) Since $\triangle ACE \sim \triangle BDE$ by (a), we have

$$\frac{\text{Area}(BDE)}{\text{Area}(ACE)} = \left(\frac{15}{25}\right)^2.$$

This gives $\text{Area}(BDE) = \left(\frac{15}{25}\right)^2 \cdot \frac{60 \cdot 25}{2} = 270 \text{ cm}^2$.

9. (a) Solving $\frac{12+k+16}{12+k+16+9+11+4} = \frac{7}{10}$ yields $k = 28$.

(b) The range, the inter-quartile range and the standard deviation are 5 books, 2 books and 1.43 books respectively.

10. (a) $f(x) = k_1 + k_2(x+4)^2$, where k_1 and k_2 are non-zero constants. $f(-3) = 0$ and $f(2) = 105$ yields

$$\begin{cases} k_1 + k_2(-3+4)^2 = 0 \\ k_1 + k_2(2+4)^2 = 105. \end{cases}$$

Solving gives $k_1 = -3$ and $k_2 = 3$. Hence $f(0) = -3 + 3 \cdot 4^2 = 45$.

(b)(i) y-intercept of G is 48.

(b)(ii) Solving $-3 + 3(x+4)^2 + 3 = 0$ yields the x-intercept of G is -4.

11. (a) The mean $= \frac{1 \cdot 15 + 2 \cdot 9 + 3 \cdot 2 + 4 \cdot 5 + 5 \cdot 4 + 6 \cdot 2 + 7 \cdot 5}{15+9+2+5+4+2+5} = 3$

(b) Median is 2 and the mode is 1, therefore the median and the mode are NOT equal.

(c)(i) $n = 42$.

(c)(ii) $n = 11$.

(c)(iii) $n = 10$.

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12. (a) By division algorithm, $p(x) = (x^2 + x + 1)(2x^2 - 37) + cx + c - 1$. Using $p(5) = 0$ yields

$$0 = (5^2 + 5 + 1)(2 \cdot 5^2 - 37) + 5c + c - 1.$$

This gives $c = -67$.

(b) $p(-3) = (9 - 3 + 1)(2 \cdot 9 - 37) - 67(-3) - 67 - 1 = 0$. Hence $x + 3$ is a factor of $p(x)$.

(c) $p(x) = (x - 5)(x + 3)(2x^2 + 6x + 7)$. Since $(-6)^2 - 4(2)(7) < 0$, $2x^2 + 6x + 7 = 0$ has no real roots and therefore the claim is NOT correct.

13. (a) $OG = \sqrt{6^2 + 8^2} = 10$.

(b) Since $OG = 10 < 13 = \sqrt{6^2 + 8^2 - (-69)} =$ radius of C. O lies inside C.

(c) The locus of P is the perpendicular bisector of OG. Let Q be the midpoint of OG, then the Pythagorean theorem yields $\frac{MN}{2} = MQ = \sqrt{13^2 - 5^2} = 12$. Thus the area of

$$OMGN = \frac{MN \cdot OG}{2} = \frac{10 \cdot 24}{2} = 120.$$

14. (a) Let r cm be the base radius of Y.

$\dfrac{\pi 24 r^2}{3} = 800\pi$ gives $r = 10$ and hence the base radius of Y is 10 cm.

(b) Volume of $Z = 800\pi + \pi \cdot 10^2 \cdot 20 = 2800\pi$ cm^3.

Since

$$\frac{\text{Volume of Z}}{\text{Volume of Y}} = \frac{2800}{800} = \frac{7}{2} \neq 8 = \left(\frac{\text{Base radius of Z}}{\text{Base radius of Y}}\right)^3,$$

Y and Z are NOT similar.

(c) Let S_X cm^2, S_Y cm^2 and S_Z cm^2 be the curved surface areas of X, Y and Z respectively.

$S_X = 2\pi(10)(20) = 400\pi$, $S_Y = \pi(10)\sqrt{10^2 + 24^2} = 260\pi$, and $S_Z = \pi(20)\sqrt{20^2 + 21^2} = 580\pi$. Since $S_X + S_Y = 660\pi > 580\pi$, I agree.

15. (a) $10! = 3628800$.

(b) Required probability $= \dfrac{\binom{8}{3} \cdot 3! \cdot 7!}{10!} = \frac{7}{15}$.

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16. (a) L_1 passes through $(0,3)$ and $(2,6)$, two-point form yields

$$L_1: \quad y-3=\left(\frac{6-3}{2-0}\right)(x-0),$$

which gives: $L_1 : y=\frac{3}{2}x+3$.

L_1 is perpendicular to L_2 implies L_2 has slope $-2/3$. Since L_2 passes through $(2,6)$, point-slope form yields

$$y=\frac{-2}{3}x+\frac{22}{3}.$$

The system of inequalities is:

$$\begin{cases} y \leq \frac{3}{2}x+3 \\ y \leq \frac{-2}{3}x+\frac{22}{3} \\ y \geq 0 \end{cases}$$

(Equivalently)

$$\begin{cases} 3x-2y+6 \geq 0 \\ 2x+3y-22 \leq 0 \\ y \geq 0 \end{cases}$$

(b) Let $P(x,y)=8x-5y$. The boundary points are $(-2,0),(11,0)$ and $(2,6)$.

$P(-2,0)=-16, P(11,0)=88$ and $P(2,6)=-14$.

Hence the least value of $P(x,y)=8x-5y$ is -16.

17. (a) Let d be the common difference.

$$\begin{cases} A(5)=A(1)+4d=26 \\ A(12)=A(1)+11d=61. \end{cases}$$

Solving gives: $A(1)=6$ and $d=5$. Hence $A(1)=6$.

(b)

$$\begin{aligned} \log_8(G(1)G(2)\ldots G(k)) &= \frac{\log_2(G(1)G(2)\ldots G(k))}{\log_2 8} \\ &= \frac{\log_2 G(1)+\log_2 G(2)+\cdots+\log_2 G(k)}{3} \\ &= \frac{A(1)+A(2)+\cdots+A(k)}{3} \\ &= \frac{(2A(1)+(k-1)d)k}{6} \\ &= \frac{5}{6}k^2+\frac{7}{6}k \end{aligned}$$

Solving $\frac{5}{6}k^2+\frac{7}{6}k<999$ gives $1 \leq k < 33.93076667$. (Note that -35.33076667 is rejected since $k>0$) Therefore the greatest value of k is 33.

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18. (a) $\dfrac{CD}{\sin(50)} = \dfrac{45}{\sin(70)}$ implies $CD \approx 36.7$ cm.

(b)(i) Construct F on AD such that $AD \perp CF$.
$DF = CD\cos\angle CDF \approx 12.54678189$ cm
$DE = DF + EF = DF + BC = 12.54678189 + 40 \approx 52.54678189$ cm
$AE = AB\cos\angle BAE = 45\cos 50° \approx 28.92544244$ cm
$AD = \sqrt{AE^2 + DE^2} \approx \sqrt{28.92544244^2 + 52.54678189^2} \approx 59.98204321$ cm
$AC = \sqrt{AB^2 + BC^2} = \sqrt{AE^2 + BE^2 + BC^2} \approx \sqrt{28.92544244^2 + 34.47199994^2 + 40^2} \approx 60.20797289$ cm

$$\angle CAD = \cos^{-1}\left(\frac{AC^2 + AD^2 - CD^2}{2(AC)(AD)}\right) \approx 35.54210789°$$

(b)(ii) Construct G on CD such that $EG \perp CD$.
The angle between the planes ACD and $BCDE$ is $\angle AGE$.
$EG = DE\sin\angle EDG = 52.54678189\sin 70° = 49.37782319$ cm

$$\angle AGE = \tan^{-1}\left(\frac{AE}{EG}\right) \approx \tan^{-1}\left(\frac{28.92544244}{49.37782319}\right) \approx 30.36169731°.$$

Therefore, the angle between the plane ACD and the plane $BCDE$ is $30.36169731°$, which exceeds $30°$.

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19. (a)

$$\begin{aligned} f(x) &= x^2 - 12kx - 14x + 36k^2 + 89k + 53 \\ &= (x - (6k+7))^2 + (5k+4) \end{aligned}$$

Therefore, the coordinates of Q is $(6k+7, 5k+4)$.

(b) $R = (7 - 6k, 5k+4)$

(c)(i) Slope of $QS = \dfrac{(5k+4) - (4-3k)}{(6k+7) - 7} = \dfrac{4}{3}$, using point-slope form yields equation of QS:

$$y - (4 - 3k) = \frac{4}{3}(x - 7)$$

$\ell_{QS} : 4x - 3y - 9k - 16 = 0$

(c)(ii)
Let U be the centre of C.
Note that QR is a horizontal line, also $QS = RS$.
Thus, the x-coordinate of U is the same as that of S, which is 7.
Let M be the midpoint of QR.
$\angle SQM = \tan^{-1}(\dfrac{MS}{MQ}) = \tan^{-1}(\dfrac{(5k+4) - (4-3k)}{(6k+7) - 7}) \approx 53.13010235°$
$\angle UQM = \dfrac{1}{2}\angle SQM \approx 26.56505118°$
$UM = MQ \tan \angle UQM \approx 6k \tan 26.56505118° = 3k$
Note that UM is the radius of C. Thus, y-coordinate of U is $(5k+4) - (3k) = 2k + 4$ and this gives the coordinates of U as $(7, 2k+4)$.
Therefore, the equation of C is

$$(x - 7)^2 + (y - (2k+4))^2 = 9k^2$$

(c)(iii) Note that $ST \perp UT$. If $STUV$ is a rectangle, UV has to be parallel to ST. Equating the slopes of UV and ST yields

$$\frac{(2k+4) - (-14)}{7 - (-29)} = \frac{4}{3}$$

Solving gives $k = 15$.

We now show that when $k = 15$, $STUV$ is indeed a rectangle.
When $k = 15$, the coordinates of U and S are $(7, 34)$ and $(7, -41)$ respectively.
Slope of $UV = \dfrac{34 - (-14)}{7 - (-29)} = \dfrac{4}{3}$
Slope of $VS = \dfrac{-41 - (-14)}{7 - (-29)} = \dfrac{-3}{4}$
Slope of ST = Slope of $QS = \dfrac{4}{3}$
Slope of $UT = \dfrac{-1}{\text{Slope of } ST} = \dfrac{-3}{4}$
Since (Slope of UV)(Slope of VS) = (Slope of VS)(Slope of ST) = (Slope of ST)(Slope of TU) = (Slope of TU)(Slope of UV) = -1,
thus, $UV \perp VS$, $VS \perp ST$, $ST \perp TU$ and $TU \perp UV$.
Therefore, when $k = 15$, $STUV$ is indeed a rectangle. i.e. it is possible.

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Dr. Koopa Koo — 全球 Top 5 壓倒哈佛 (Harvard University)*

頂級學歷 · 非凡成就

- 美國華盛頓大學 (University of Washington) 數學博士 (PhD in Mathematics)
- 在被 ≪Time Magazine (時代雜誌)≫ 譽為全球最難的數學競賽”William Lowell Putnam Mathematics Competition” 中，於第 63 屆擊敗來自全球 3349 位頂尖數學高手勇奪第 35 名，獲頒發榮譽獎。

教學經驗 · 全球頂尖

- 受邀於以下世界頂尖大學發表學術研究演講：哈佛大學 (Harvard University)、華盛頓大學 (University of Washington) 費爾茲數學研究所 (Fields Institute of Mathematics)、加州大學柏克萊分校研究中心 (UC Berkeley, MSRI)。
- 亦獲邀於本地三大頂尖學府：香港大學、香港中文大學及香港科技大學演講，聽眾包括中學生、大學生、碩士生、博士生，甚至大學教授皆為座上客
- 國際數學奧林匹克 (International Mathematical Olympiad，IMO) 金牌教練
- 恆隆數學獎金獎顧問

戰績封神 · 市場領導

- DSE 數學科市場領導，被同學公認為補習界「數學之神」，連數底最差的學生都能輕易得救。
- 超過 7000 位門生於數學科奪 5**/5*/5, 備有成績單副本作實，戰績橫掃全港
- 全港最多學生選報數學科名師
- 全港最多學生奪 5**/5*/5 數學科名師
- 全港最高 5**/5*/5 比率數學科名師

*Top Institutions in Mathematics (Times Higher Education, 2011)

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口碑傳承 · 家長推崇

- 2007–2021 連續 14 年均有門生入讀世界頂尖學府如哈佛大學 (Harvard University) 、麻省理工大學 (Massachusetts Institute of Technology，MIT)、普林斯頓大學 (Princeton University)、劍橋大學 (Cambridge University)、牛津大學 (University of Oxford) 等。
- 每年皆有眾多的門生考入三大「神科」，如醫科、法律、環球商業等。
- 不少數學底子較弱的同學於報讀 Dr. Koopa Koo 的課程後皆有飛躍的進步，高考純數由 U 變 A 及文憑試由 3 變 5** 的例子不計其數，口碑傳承。
- 課程專業質素一直深受家長擁戴，甚至補習界同業亦將子女的前途交託於 Dr. Koopa Koo 手中。

獲獎無數 · 蜚聲國際

- 獲美國華盛頓大學 (University of Washington) 頒發頂級學者獎
- 獲美國華盛頓大學 (University of Washington) 頒發全額獎學金修讀博士課程，獎學金額超過 HKD $2,000,000$
- 於大學期間，為美國哈佛大學 (Harvard University) 代數組合研究組成員
- 為 Boston College 數學系第一個本科生獲頒數學系教育獎學金，於數學系任助教一職
- 為 Boston College 首位兩奪高等研習基金，表揚在數學研究的突出表現
- 為 Boston College 數學系全系第一學生
- 獲 Boston College 頒發 Dean’s Scholar，為全級最優秀學生

國際學者 · 數學權威

- 博士論文研究的領域，岩澤理論於証明 350 年難題「費馬大定理」中，起了決定性的作用。
- 積極參與學術研究，並於頂尖的數論期刊中發表學術研究論文，擁有自己的數學定理。

經驗豐富 · 貢獻不斷

- 獲邀於學友社在 2012 大學聯招講座向高中同學分享中學文憑試數學必修科的應試心得。
- 獲香港電台及科技大學民間電台邀請與應屆考生分享有關數學必修科和兩個選修單元的應試心得。
- 於美國華盛頓大學 (University of Washington) 任職講師，教授高等微積分、多變量微積分、線性代數及線性分析等工程系必修課程
- 於美國華盛頓大學 (University of Washington) 任客席講師教授數學博士生代數數論

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獨門秘技 · 秒殺試題

- 「Koopa 神技」將重新定義何謂考試技巧。
- 獨創一系列解題方法，被同學譽為「Koopa 神技」，易學易用。，難怪有同學說:「Koopa’s way is the winning way!」
- 筆記內容主力提升數學概念、解題技巧，以及針對破解 HKDSE 設題者思路，令同學輕鬆「秒殺」HKDSE。

概念封神 · 輕鬆滿分

- 除了幫助同學清 Concept，更重要是教同學用 Concept。教同學應該點樣用？邊度用？幾時用？
- 獨門「萬伏心經」練成後當萬伏不侵，扣分零可能。以後只有你伏人，而人不能伏你。

用心教學 · 萬人頌讚

實力技巧皆封神
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